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PLASTIC DEFORMATIONS OF A CYLINDRICAL SHELL UNDER THE ACTION OF A PLANAR EXPLOSION WAVE

R. G. Yakupov

UDC 539.374

A thin-walled circular cylindrical shell of infinite length is located in the ground. At a specified distance from the shell a planar charge of explosive material, of infinite length along the direction of the cylinder axis, explodes, and a planar plastic shock wave develops in the medium. The wave front is parallel to the cylinder directrix, and the wave parameters are known. It is necessary to find the residual cylinder deformations as a function of explosion wave pressure.

We will locate the origin of a coordinate system y, w at the point O in the direction of the incident wave (Fig. 1a). We write the equations of motion of a shell element experiencing displacements of the order of the magnitude of the shell wall thickness in the form [1]

$$T' = N_y' = 0,\ M'' + ((1/\mathrm{R}) + w'')N_y + q + q_1 - \rho H\ddot{w} = 0, \tag{1.1}$$

where T, N_y are the tangent and normal stresses in the mean surface; M_y, bending moment in the peripheral direction; R, H, radius and wall thickness of the shell; ρ, density of the material; w, radial displacement; q, q_1, pressure of the wave and the surrounding medium; the prime denotes differentiation with respect to y, and the dot, with respect to t.

Reflection of a planar plastic shock wave from a planar barrier at normal incidence and incidence at an angle has been studied in [2, 3]. To determine the explosion wave pressure on the shell, we will use the results of those studies and the "isolated element principle," according to which an incident plane wave is reflected from a curvilinear boundary in the vicinity of each point just as it is reflected from a small element of a plane passing through the given point. We write the expression for wave pressure in the form

$$q = p_0(1 - t/t_0)\cos\theta,\ -\pi/2 \leqslant \theta \leqslant \pi/2,\ t > 0,\ q = 0, \quad \pi/2 \leqslant \theta \leqslant -\pi/2, \tag{1.2}$$

where $p_0 = p_{1*}(1 + \sqrt{n})$; p_{1*} is the pressure on the incident wave front at the moment of

Ufa. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 4, pp. 127-132, July-August, 1982. Original article submitted June 26, 1981.

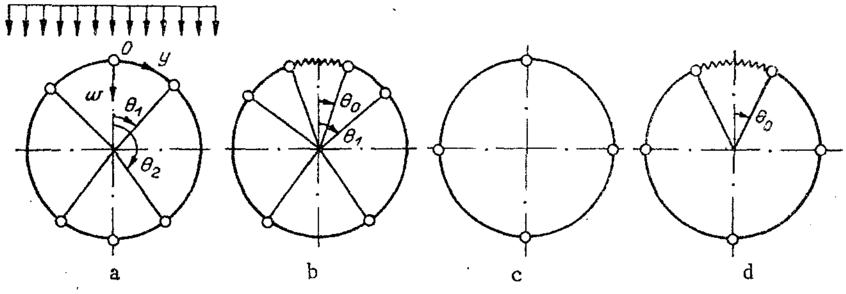

Fig. 1.

reflection; t_0 is the period over which the pressure acts; n is the medium's compression exponent; and $\theta = y/r$ is the angular coordinate.

On the side opposite the incident wave, due to deformations and displacement as a rigid cylinder, the shell experiences a pressure which we define by [4]

$$q_1 = p_c + \rho_0 c_1 \dot{w}, \ \pi/2 \leqslant \theta \leqslant -\pi/2, \ q_1 = 0,$$
$$-\pi/2 \leqslant \theta \leqslant \pi/2, \tag{1.3}$$

where p_c, ρ_0 are the yield point and density of the medium; $c_1^2 = E_1/\rho_0$; E_1 is the plastic modulus.

The total displacement of the shell is equal to the sum of the displacements of an absolutely rigid cylinder u and the displacement produced by deformation w_0. Planar motion of a rigid cylinder under the action of a pressure wave is described by the equation

$$\pi m_0 \ddot{u} + \int_0^{\pi/2} (\varkappa_0 \dot{w} - q) \cos\theta d\theta + p_c = 0.$$

Here we assume that the pressure p_c is uniformly distributed over the length of the semicircle $\dot{w} = \dot{u} \cos\theta$, $m_0 = \rho H$, $\varkappa_0 = \rho_0 c_1$.

2. Let the shell material be ideally rigid-plastic. Below we will use a piecewise-linear approximation of the plasticity condition for thin-walled shells [5], shown in Fig. 2 by the dashed line, and described by

$$|N_y/N_0| \leqslant 1, \ |M_y/M_0| \leqslant 1,$$

where $N_0 = \sigma_y H$; $M_0 = \sigma_y H^2/4$; $\sigma_y = 2\sigma_s/\sqrt{3}$; σ_s is the yield point for uniaxial deformation.

We will consider a motion mechanism, which we will term mechanism I, in which in the direction of the incident wave at the points $\theta = 0, \pm\theta_1$, there appear three plastic joints (Fig. 1a). The segments of the envelope between the plastic joints are in a plastic state AD. The mean surface then experiences compression. The flow law corresponding to this limiting state can be written in the form

$$\dot{\varepsilon}_y : \dot{\varkappa}_y = -1 : 0 \ (N_y = -N_0; -M_0 \leqslant M_y \leqslant M_0),$$

where $\dot{\varepsilon}_y$ and $\dot{\varkappa}_y$ are generalized deformation velocities, corresponding to generalized stresses N_y and M_y. At the angular points A and D flow mechanisms such as $\dot{\varepsilon}_y : \dot{\varkappa}_y = -1 : 0$ and $\dot{\varepsilon}_y : \dot{\varkappa}_y = (1 - \lambda):\lambda$ are also possible.

Thus, at the limits of each segment the field of displacement velocities satisfying the plasticity condition and the flow law is a solution of the equation $\dot{\varkappa}_y = \dot{w}'' = 0$ and is defined by

$$\dot{w}_0 = C_1(t)y + C_2(t). \tag{2.1}$$

We find the constants C_1 and C_2 from the conditions $\dot{w}_0 = \dot{w}_1 (y = 0)$, $\dot{w}_0 = 0 (y = y_1)$:

$$C_1 = \mp \dot{w}_1/y_1 \ (0 \leqslant y \leqslant y_1; \ -y_1 \leqslant y \leqslant 0), \ C_2 = \dot{w}_1.$$

The total velocity field is described by $\dot{w} = \dot{w}_0 + \dot{u} \cos\theta$.

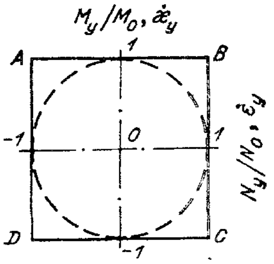

Fig. 2.

Using this stress field and flow law, we transform Eq. (1.1) to

$$M'' - N_0/R + q + q_1 - m_0\ddot{w}_0 = 0. \tag{2.2}$$

The solution of Eq. (2.2) must also satisfy the boundary conditions

$$M(0,\ t) = M_0,\ M'(0,\ t) = 0; \tag{2.3}$$

$$M(\pm\theta_1,\ t) = -M_0,\ M'(\pm\theta_1,\ t) = 0. \tag{2.4}$$

We denote by M_1 the bending moment in the region $\theta_1 \leqslant \theta \leqslant \theta_1$. Substituting in Eq. (2.2) the equations (1.2), (2.1), taking $q_1 = 0$, and satisfying conditions (2.3), we find

$$\frac{M_1}{R^2} = \frac{q_3}{2} + \frac{A_1}{2}\theta_1^2 + \frac{m_0\dot{C}_1 R}{6}\theta^3 - p_0\varphi(1-\cos\theta), \tag{2.5}$$

where $q_2 = N_0/R;\ q_3 = 2M_0/R^2;\ \ A_1 = q_2 + m_0\dot{C}_2;\ \varphi = 1 - t/t_0$.

Applying to Eq. (2.5) the boundary conditions (2.4), we obtain a system of equations for the displacements

$$\frac{m_0\theta_1^2}{3}\ddot{w}_1 = p_0\varphi(1-\cos\theta_1) - q_3 - \frac{q_2}{2}\theta_1^2,\quad \frac{m_0\theta_1}{2}\ddot{w}_1 = p_0\varphi\sin\theta_1 - q_2\theta_1. \tag{2.6}$$

In Eq. (2.6) we set $\ddot{w}_1 = t = 0$ and find the expressions for the limiting static pressure corresponding to mechanism I. The limiting static pressure will be the lesser of the pressures defined by

$$p_s = (q_2\theta_s^2 + 2q_3)/[2(1-\cos\theta_s)],\quad p_s = q_2\theta_s/\sin\theta_s. \tag{2.7}$$

Here θ_s is the angular coordinate of the plastic cylinder defined by the solution of the equation

$$\theta_s[2(1-\cos\theta_s) - \theta_s\sin\theta_s] - (H/R)\sin\theta_s = 0.$$

After eliminating the acceleration from system (2.6) we find an expression for the coordinate θ_1 as a function of wave pressure at the moment of reflection:

$$p_0 = (6q_3 - q_2\theta_1^2)/\{2[3(1-\cos\theta_1) - 2\theta_1\sin\theta_1]\}. \tag{2.8}$$

Solution (2.5) does not contradict the original assumptions as to the stress field with $M_1''(0,\ t) \leqslant 0,\ \ M_1''(\pm\theta_1,\ t) \geqslant 0$, whence it follows that at the initial moment it must be true that

$$p_0 \leqslant p_{01},\ p_0 \leqslant p_{02}, \tag{2.9}$$

where $p_{01} = q_2\theta_1/(2\sin\theta_1 - \theta_1)$; $p_{02} = q_2/\cos\theta_1$.

We will assume that there are also formed in the lower half of the shell at points with coordinates $\pm\theta_2$ joints (Fig. 1a), with the velocity field having the form

$$\dot{w} = C_3 y + C_4 + \dot{u}\cos\theta, \tag{2.10}$$

where the integration constants are equal to

$$C_3 = \pm[\dot{w}_2/(\pi - \theta_2)R](\theta_2 \leqslant \theta \leqslant \pi;\ -\pi \leqslant \theta \leqslant -\theta_2),$$

$$C_4 = -\dot{w}_2\theta_2/(\pi - \theta_2)$$

and are determined from the conditions $\dot{w}_0 = 0(\theta = \pm\theta_2)$, $\dot{w}_0 = \dot{w}_2(\theta = \pm\pi)$. The motion described by Eq. (2.10) will be called mechanism Ia. Substituting in Eq. (2.2) Eqs. (1.3), (2.10) and taking $q = 0$, we perform the integration. The bending moment in the region

$\theta_2 \leqslant \theta \leqslant -\theta_2$ will be termed M_2. Applying to Eq. (2.2) the conditions $M_2(\pm\pi, t) = M_0$, $M_2'(\pm\pi, t) = 0$, we find M_2. Then with the aid of the condition $M_2(\pm\theta_2, t) = -M_0$, $M_2'(\pm\theta_2, t) = 0$ we obtain the system

$$[(\pi - \theta_2)^2/3](m_0\ddot{w}_2 - \varkappa_0\dot{w}_2) = -q_3 - [(\pi - \theta_2)^2/2] \times (q_2 - p_c) - \varkappa_0\dot{u}(1 + \cos\theta_2), \tag{2.11}$$

$$[(\pi - \theta_2)/2](m_0\ddot{w}_2 - \varkappa_0\dot{w}_2) = -(\pi - \theta_2)(q_2 - p_c) - \varkappa_0\dot{u}\sin\theta_2.$$

Eliminating the velocity and acceleration from Eq. (2.11) we find an expression for the coordinate of the plastic joints θ_2. At the initial moment we find θ_2 from

$$\theta_2 = \pi - \sqrt{6q_3/(q_2 - p_c)} \approx \pi - \sqrt{3H/R}.$$

The conditions $M_2''(\pm\pi, t) \leqslant 0$, $M_2''(\pm\theta_2, t) \geqslant 0$ give inequalities which take on the form $q_2 - p_c \geqslant 0$ at the initial moment and are always satisfied.

3. We will consider motion of the shell, when the region $\mathrm{I}(-\theta_0 \leqslant \theta \leqslant \theta_0)$ is in a completely plastic state A, while the region II $(\theta_0 \leqslant \theta \leqslant \theta_1, -\theta_1 \leqslant \theta \leqslant -\theta_0)$ is in a plastic state AD and the velocity fields have the respective forms

$$\dot{w} = \dot{w}_0 + \dot{u}\cos\theta, \quad \dot{w} = C_5 y + C_6 + \dot{u}\cos\theta, \tag{3.1}$$

where $C_5 = \pm\dot{w}_0(\theta_0)/[(\theta_0 - \theta_1)R]$; $C_6 = -\dot{w}_0(\theta_0)\theta_1/(\theta_0 - \theta_1)$; $\theta_0 = \theta_0(t)$ is the unknown angular coordinate of the boundary between the regions with differing plastic regimes. The motion described by Eq. (3.1) will be termed mechanism II.

The equation of motion for region I has the form

$$m_0\ddot{w}_0 = p_0\varphi\cos\theta - q_2. \tag{3.2}$$

We note that Eq. (3.2) follows from the moment-free theory of a cylindrical shell.

We integrate the equation of motion of region II, substituting in Eq. (2.2) the second expression of Eq. (3.1) and taking $q_1 = 0$. Applying the conditions $M_{11}(\pm\theta_0, t) = M_0$, $M_{11}(\pm\theta_1, t) = -M_0$, $M_{11}'(\pm\theta_0, t) = 0$, $M_{11}'(\pm\theta_1, t) = 0$ to the solution, we arrive at

$$\frac{m_0 f_1}{3}[f_1\ddot{w}_0(\theta_0) + \dot{\theta}_0\dot{w}_0(\theta_0)] = p_0\varphi f_2 - \frac{q_2}{2}f_1^2 - q_3, \tag{3.3}$$

$$\frac{m_0}{2}[f_1\ddot{w}_0(\theta_0) + \dot{\theta}_0\dot{w}_0(\theta_0)] = p_0\varphi f_3 - q_2 f_1,$$

where $f_1 = \theta_1 - \theta_0$; $f_2 = \cos\theta_0 - \cos\theta_1 - f_1\sin\theta_0$; $f_3 = \sin\theta_1 - \sin\theta_0$.

The limiting static pressure and angular coordinate of the plastic joints corresponding to mechanism II are defined by

$$p_s = (q_2 f_1^2 + 2q_3)/2f_2, \quad p_s = q_2 f_1/f_3, \quad p_s = q_2/\cos\theta_0.$$

In analogy to Eq. (2.8) we find an expression for the coordinate θ_0 as a function of wave pressure

$$p_0 = (q_2 f_1^2 - 6q_3)/(4f_1 f_3 - 6f_2). \tag{3.4}$$

From the condition $M_{11}''(\pm\theta_0, t) \leqslant 0$, $M_{11}''(\pm\theta_1, t) \geqslant 0$ we find inequalities which limit the pressure value at the moment of reflection

$$p_0 \leqslant p_{03}, \quad p_0 \leqslant p_{04}, \tag{3.5}$$

where $p_{03} = (q_2 f_1^2 + 6q_3)/[2(3f_2 - f_1^2\cos\theta_0)]$; $p_{04} = q_2/\cos\theta_1$.

We can divide the shell motion into three stages. The first is the drive stage, $0 \leqslant t \leqslant t_1$. At the end of the first stage the displacement velocity reaches its highest value. We determine t_1 from the equation $\ddot{w}_0(\theta, t_1) = 0$. The value of θ_0 is time independent, but does depend on p_0, as given by Eq. (3.4).

The second stage, $t_1 \leqslant t \leqslant t_2$. In the second stage the kinetic energy and displacement rate of the plastic zone decrease. We find the inflection and the angle $\theta_0(t)$ from the solution of system (3.2), (3.3) with initial conditions $\theta_0(t_1) = \theta_0$, $\dot{w}_0(\theta_0, t) = \dot{w}_0(\theta_0, t_1)$, $w_0(\theta_0, t) = w_0(\theta_0, t_1)$. The value of the coordinate θ_0 decreases to zero. The time t_2 is determined from $\theta_0(t_2) = 0$.

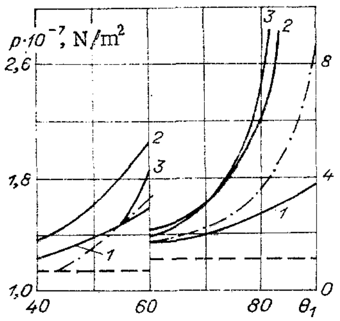

Fig. 3.

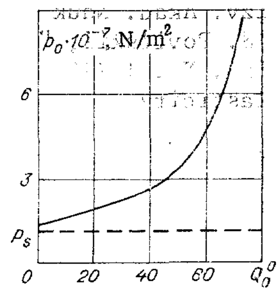

Fig. 4.

The third stage $t_2 \leqslant t \leqslant t_3$. Analysis of this stage is analogous to that of the shell motion in Sec. 2. The time t_3 is determined from the condition of equality to zero of the deflection velocity.

Shell motion is not limited to the mechanisms considered above. Other kinematic states are possible, which can be studied in the same manner. We have considered additional motion mechanisms III and IV. In mechanism III, plastic joints appear at four diametrically opposed points (Fig. 1c). The shell deforms similarly to the lowest order form with elastic deflection, and motion of four quarters rigid with respect to deflection in plastic state AD occurs. In the case of mechanism IV the region $-\theta_0 \leqslant \theta \leqslant \theta_0$ in the direction of the incident wave is in plastic state A, and the region $\theta_0 \leqslant \theta \leqslant \pi/2, -\pi/2 \leqslant \theta \leqslant -\theta_0$ is in plastic state AD.

4. A shell has dimensions H = 0.015 m, R = 0.5 m, $\sigma_S = 3 \cdot 10^8$ N/m², and is located in water-unsaturated sandy soil of disrupted structure with frame density of $1.7 \cdot 10^3$ kg/m³, moisture content $w^0 = 4$-8%. The compression diagram has the form $p = 3.35 \cdot 10^5 \varepsilon^{2.5}$ [6], where ε is the relative deformation of the soil. We take the limiting compression of the soil as $\varepsilon_+ = 0.2$. An explosive charge has a thickness $2a = 0.002$ m, with isentropy index of the detonation products $\gamma = 1.25$. The initial pressure in the cavern is $2 \cdot 10^9$ N/m². We will define the wave parameters as in [7].

From comparison with the minimum pressures p_S of all four motion mechanisms it follows that the smallest limiting static pressure is $p_S = 115 \cdot 10^5$ N/m², corresponding to mechanism I. Therefore in the case of static pressure action, according to kinematic theory, mechanism I is realized.

With dynamic loading shell deformations commence at pressures $p_0 > p_S$, where the pressure on the incident wave front $p_{1*} > 45 \cdot 10^5$ N/m², which corresponds to explosion distances of 8 m or less.

Figure 3 shows graphs of the pressures $p_{0i}(i = 1-4)$ as a function of angle θ_1. The graph scale up to $\theta_1 = 60°$ is shown on the left, while for $\theta_1 > 60°$, the right-hand scale is valid. Curves 1-3 are graphs of $p_{01}(\theta_1)$, $p_{02}(\theta_1)$, $p_{03}(\theta_1)$, $p_{04}(\theta_1) = p_{02}$. The dash–dot curve shows the change in p_0 with θ_1 according to Eq. (2.8). The horizontal dashed line is the pressure p_S. The joint coordinates are $\theta_S = 44°$, $\theta_2 = \pm 163°$.

As is evident from Fig. 3, in the pressure range $p_s \leqslant p_0 \leqslant 147 \cdot 10^5$ N/m² inequality (2.9) is satisfied and mechanism I is realized. At $147 < p_0 \leqslant 860 \cdot 10^5$ N/m² inequality (3.5) is satisfied and shell deformation follows mechanism II. On the side opposite the incident wave, mechanism Ia is realized. A curve of $p_0(\theta_0)$ is shown in Fig. 4. For the case $p_0 \geqslant 860 \cdot 10^5$ N/m² the angle $\theta_1 = \pi/2$, and the shell deforms by mechanism IV. Mechanism III is not realized, since the inequalities analogous to Eqs. (2.9), (3.5) are not fulfilled.

From solution of the first equation of Eq. (2.6) we find the residual deflection, which at a pressure of $p_0 = 132 \cdot 10^5$ N/m² and $c_1 = 100$ m/sec is equal to 1.64 cm. The coordinate of the deformation region $\theta_1 = 50°$, and the explosion distance is 7.5 m.

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EQUATIONS OF ELASTOVISCOPLASTIC MEDIUM WITH FINITE DEFORMATIONS

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UDC 539.371

In this paper, we examine the nonstationary equations of the theory of flow of finitely deformed elastoviscoplastic materials. We analyze two approaches to describing the kinematics of such media. We study the restrictions imposed on the determining equations by the entropy inequality and the requirements of invariance relative to orthogonal transformations of the actual, unloaded and initial configurations. The complete system of equations is written in divergence form, which permits obtaining all allowable relations at strong discontinuities. In the adiabatic approximation, the system of equations reduces to a symmetrical form and we formulate sufficient conditions for hyperbolicity.

1. Kinematics. Let ξ be the radius vector of a particle in the medium in the initial configuration of the body and $\mathbf{x}$ the actual instantaneous configuration. We shall assume that the initial configuration is the natural configuration [1, 2] with constant temperature $\Theta = \Theta_0$ and density $\rho = \rho_0 = \text{const}$. We shall denote by $\overset{\circ}{e}_i, \hat{e}_j$ the basis vectors of the starting and accompanying Lagrangian system of coordinates [1] and by e_k the basis of the spatial Cartesian coordinate system, such that

$$d\boldsymbol{\xi} = d\xi^i \overset{\circ}{e}_i,\quad d\mathbf{x} = dx^k e_k = d\xi^j \hat{e}_j. \tag{1.1}$$

We shall assume that the mapping (deformation) of the starting configuration into the actual configuration

$$\mathbf{x} = \mathbf{x}(\xi, t), \tag{1.2}$$

where t is the time, is mutually unique and continuously differentiable the required number of times. For fixed t, it follows from (1.2) that

$$d\mathbf{x} = \mathbf{F}\cdot d\boldsymbol{\xi} = \left(\overset{\circ}{e}_a F^{a\cdot}_{\cdot b}\overset{\circ}{e}^b\right)\cdot\left(\overset{\circ}{e}_j d\xi^j\right) = \overset{\circ}{e}_a F^{a\cdot}_{\cdot j} d\xi^j, \tag{1.3}$$

where $\mathbf{F}$ is the tensor of the gradient of the total deformation. Equating (1.1) and (1.3) we see that

$$\hat{e}_j = \overset{\circ}{e}_i F^{i\cdot}_{\cdot j}, \tag{1.4}$$

i.e., the matrix $F^{i\cdot}_{\cdot j}$ is the linear transformation of both $d\xi$ into dx and the basis $\overset{\circ}{e}_i$ into the basis $\hat{e}_j$.

Using the definition of the velocity vector $\mathbf{v} = \partial\mathbf{x}(\xi, t)/\partial t|_\xi$ and relation (1.4), we obtain

$$d\mathbf{v} = \nabla\mathbf{v}\cdot d\mathbf{x} = d\xi^i \frac{\partial \hat{e}_i}{\partial t}\bigg|_{\xi^m} = d\xi^i \frac{\partial F^{k\cdot}_{\cdot j}}{\partial t}\bigg|_{\xi^m} \overset{\circ}{e}_k = \frac{\partial \mathbf{F}}{\partial t}\bigg|_{\xi}\cdot d\boldsymbol{\xi},$$

from where in view of the arbitrariness of $d\mathbf{x}$ follows the kinematic relation [1, 2]

Moscow. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 4, pp. 133-139, July-August, 1982. Original article submitted April 20, 1981.

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